B.Sc. Eng. (ME/IPE)/1st Sem. **Date: January 16, 2025**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF NATURAL SCIENCES

Mid-Semester Examination **Winter Semester**: 2024-2025
Course Number: MATH 4111 **Full Marks**: 120
Course Title: Modelling with calculus and ODE **Time**: 2 Hours

Please answer according to the order of the questions. Answer all the **4 (FOUR)** questions. The symbols have their usual meanings. Marks of each question and the corresponding CO and PO are written in the brackets.

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1. a) A 25-foot ladder is leaning against a house in Fig. Q1(a) . If the base of the ladder is pulled away from the house at a rate of 2 feet per second, then show that the top will move down the wall at a rate of **(10) (CO1) (PO1)**

$$r = \frac{2x}{\sqrt{625 - x^2}} \text{ ft/sec}$$

where x is the distance between the base of the ladder and the house, and r is the rate in feet per second.
(i) Find the rate r when x is 15 feet.
(ii) Find the limit of r as x approaches 25 from the left.

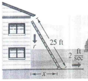

Fig. Q1(a)

b) The roof of the house in Fig. Q1(b) is considered a function $f(x)$. **(10) (CO1) (PO1)**
(i) Formulate the pricewise function $f(x)$.
(ii) Test whether the function $f(x)$ is continuous at $x = 0$ or not.
(iii) Examine whether the function $f(x)$ is differentiable at $x = 0$ or not.
(iv) Discuss with figures when a function is not differentiable.

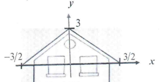

Fig. Q1(b)

c) (i) State Leibnitz's Theorem for nth order derivative. **(10) (CO1) (PO1)**
Apply Leibnitz's Theorem to find $\frac{d^4 y}{dx^4}$, $if\ y = e^{-2x}\sin 3x$.
(ii) Find y_n if $y = \cos(ax + b)$.

2. a) An airplane is flying on a flight path that will take it directly over a radar tracking station, as shown in Figure Q2(a). The distance s is decreasing at a rate of 400 miles per hour when $s = 10$ miles. Apply suitable technique to find the speed of the plane. **(10) (CO2) (PO1)**

Fig. Q2(a)

b) State Rolle's theorem and Mean Value Theorem (MVT).
Two stationary patrol cars equipped with radar are 5 miles apart on a highway, as shown in Fig. Q2(b). As a truck passes the first patrol car, its speed is clocked at 55 miles per hour. Four minutes later, when the truck passes the second patrol car, its speed is clocked at 50 miles per hour. Apply MVT to show that the truck must have exceeded the speed limit (of 55 miles per hour) at some time during the 4 minutes. **(10) (CO2) (PO1)**

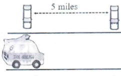

Fig. Q2(b)

c) The Hubble Space Telescope was deployed on April 24, 1990, by the space shuttle Discovery. A model for the velocity of the shuttle during this mission, from liftoff at $t = 0$ until the solid rocket boosters were jettisoned at $t = 2$ minutes, is given by **(10) (CO2) (PO1)**

$$v(t) = 0.001302t^3 - 0.09029t^2 + 23.61t - 3.083$$

(in feet per second). Apply this model, to estimate the absolute maximum and minimum values of the acceleration of the shuttle between liftoff and the jettisoning of the boosters.

3. a) Define linear differential equations. Determine the order, degree, of the given differential equations and state whether the equation is linear or nonlinear. **(10) (CO1) (PO1)**

(i) $(1 - y^2)\frac{d^2y}{dt^2} + t\frac{dy}{dt} + y = e^t$, (ii) $\sqrt[4]{\frac{dy}{dx} + 2x\left(\frac{d^4y}{dx^4}\right)^3} = \sqrt[3]{x-2}$,

(iii) $(x^4 + 2y^3)\frac{d^2y}{dx^2} + (3x^2 - x)\frac{dy}{dx} + y = 0$,

(iv) $\frac{d^3y}{dx^3} + x\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 2y = e^x$.

b) Find the differential equation of the family of circle of radius r with centers on the x-axis. i.e., $(x-a)^2 + y^2 = r^2$ where, a and r are arbitrary constants. **(10) (CO1) (PO1)**

c) Define exact differential equations. Solve the following differential equation **(10) (CO1) (PO1)**

$$(2x\log x - xy)dy + 2ydx = 0.$$

4. a)	Apply suitable method to solve the given differential equation $$\sin x \frac{dy}{dx} + 2y = \tan^3\left(\frac{x}{2}\right).$$	**(10)** (CO2) (PO1)
b)	An inductance of $L = 2$ henries and a resistance of $R = 20$ ohms are connected in series with an electromagnetic force. E volts. If the current is zero when $t = 0$, find the current at the end of 0.01 sec if $E = 100$ volts.	**((10)** (CO2) (PO1))
c)	A moving body is opposed by a force per unit mass of value cx and a resistance per unit mass of value bv^2, where x and v are the displacement and velocity of the particle at that instant. Formulate and solve the differential equation for the motion, and hence find the velocity of the particle in terms of x, if it starts from rest.	**((10)** (CO2) (PO1)