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B.Sc. Eng.(CSE/SWE, 1st Sem.) **Date: April 15, 2025**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
DEPARTMENT OF NATURAL SCIENCES

Semester Final Examination **Winter Semester**: 2023-2024
Course Number: Math 4141 **Full Marks**: 160
Course Title: Matrix and Differential Equations **Time**: 2 Hours

Please answer according to the order of the questions. Answer all the **4 (FOUR)** questions. The symbols have their usual meanings. Marks of each question and the corresponding CO and PO are written in the brackets.

1. (a) Consider the function $f(x)=3x^4+4x^3-12x^2+2$, then **[12]** (CO1) (PO1)

(i). find the intervals on which $f(x)$ is increasing and decreasing.

(ii). find the intervals on which $f(x)$ is concave up and concave down.

(iii). find the critical points and inflection points, if any.

(iv). sketch the graph of $f(x)$.

(b) (i). Find the relative extrema of the function $f(x)=3x^5-5x^3$ using both first and second derivative tests. **[14]** (CO2) (PO2)

(ii). Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with radius 6 inches and height 10 inches.

(c) (i). Find the absolute maximum and absolute minimum values of the function $f(x)=x^3-3x^2+1$ on the interval $\left[-\frac{1}{2},4\right]$, and determine where these values occur. **[14]** (CO3) (PO2)

(ii). Write down the statement of Rolle's Theorem. Verify Rolle's Theorem for the function $f(x)=\ln\left(4+2x-x^2\right)$ on the interval $[-1,3]$.

2. (a) (i). State and prove Euler's Theorem for a homogeneous function of three variables. Hence, verify Euler's Theorem for the function $u(x,y)=\frac{x\left(x^3-y^3\right)}{x^3+y^3}$. **[20]** (CO2) (PO2) (ii). If $u=\tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\sin 2u$.

(b) (i). Find the second-order partial derivatives of $f(x,y)=4x^2-2y+7x^4y^5$. **[10]**

(ii). Let $u(w,x,y,z)=xe^{yw}\sin^2 z$ be a given function, then find $\frac{\partial^4 u}{\partial x\partial y\partial w\partial z}$ and evaluate $\frac{\partial u}{\partial z}(0,0,1,\pi)$. (CO3) (PO2)

(c) (i). Define radius of curvature of a curve. Find the radius of curvature at the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$ of the curve $x^3+y^3=3axy$. **[10]** (CO4) (PO2)

(ii). Find the nth Maclaurin polynomial for $\frac{1}{1-x}$.

3. Consider the four points $A(3,6,9)$, $B(1,2,3)$, $C(2,3,1)$, and $D(4,6,2)$. **[40]**

(i). Determine whether AB and CD intersect or not. If so, find the point of intersection. (CO1, CO2)

(ii). Find the equation of plane in normal form through the points A, B, and C. (PO1, PO2)

(iii). Find the perpendicular distance from the point D to the plane ABC.

(iv). Find the angle between the line CD and the plane ABC.

4. (a) Find the length and equation of the shortest distance between the lines $3x-9y+5z=0=x+y-z$ and $6x+8y+3z-13=0=x+2y+z-3$. **[20]** (CO3) (PO2)

(b) Find the equations of the tangent planes to the sphere $x^2+y^2+z^2+6x-2z+1=0$ which pass through the line $\frac{16-x}{2}=\frac{z}{2}=\frac{y+15}{3}$. **[20]** (CO4) (PO2)

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