B.Sc. Engg. (SWE) 1st Semester

Library

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ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Natural Sciences (N.Sc.)

MID-SEMESTER EXAMINATION
DURATION: 2 HOURS

WINTER SEMESTER, 2024-2025
FULL MARKS: 160

MATH 4141: Geometry and Differential Calculus

Programmable calculators are not allowed. Do not write anything on the question paper.
Answer all 4 (four) questions. Marks of each question and corresponding CO and PO are written in the right margin with brackets.

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1. (a) Consider the piecewise function $g(x)$ defined as follows: **[9]** (CO1) (PO2)

$$g(x)=\begin{cases} x; & \text{if } x<1 \\ 3; & \text{if } x=1 \\ 2-x^2; & \text{if } 1<x\le 2 \\ x-3; & \text{if } x>2 \end{cases}$$

Now, evaluate each of the following limits, if exists.

(i). $\lim_{x\to 1} g(x)$.

(ii). $\lim_{x\to 2^-} g(x)$.

(iii). $\lim_{x\to 2^+} g(x)$.

(b) Define the continuity of a function. Let a function $f(x)$ be defined as follows: **[9]** (CO1) (PO1)

$$f(x)=\begin{cases} \dfrac{x^2-4}{x-2}; & x<2 \\ ax^2-bx+3; & 2\le x<3 \\ 2x-a+b; & x\ge 3 \end{cases}$$

Find the values of the constants a and b that make the function $f(x)$ continuous everywhere.

(c) (i). Find $f'(x)$ if $f(x)=\left(x^2+1\right)\left(x+5+\dfrac{1}{x}\right)$ using product rule. **[12]** (CO1) (PO2)

(ii). Find $g'(x)$ if $g(x)=\dfrac{4-3x}{3x^2+x}$ using quotient rule.

(iii). Find $\dfrac{dy}{dx}$ if $y=\left[1+\sin^3\left(x^5\right)\right]^{12}$ using chain rule.

(d) Compute the limits (if exist) using L'Hôpital's rule: **[10]** (CO1) (PO2)

(i). $\lim_{x\to 0}\dfrac{\tan x-x}{x^3}$.

(ii). $\lim_{x\to 1^+}\left(\dfrac{1}{\ln x}-\dfrac{1}{x-1}\right)$.

2. (a) (i). If $y=\left[x+\sqrt{1+x^2}\right]^m$, show that $\left(x^2+1\right)y_{n+2}+(2n+1)xy_{n+1}+\left(n^2-m^2\right)y_n=0$ using Leibnitz's theorem. **[20]** (CO1) (PO1)

(ii). If $y=e^{ax}\sin(bx+c)$, then prove that

$$y_n=e^{ax}\left(a^2+b^2\right)^{\frac{n}{2}}\sin\left[bx+c+n\tan^{-1}\left(\frac{b}{a}\right)\right].$$

(b) Use implicit differentiation technique to find $\frac{dy}{dx}$ for a Lemniscate represented by $2\left(x^2+y^2\right)^2=25\left(x^2-y^2\right)$. Hence, find the equation for the tangent line to the Lemniscate at the point $(3,1)$. **[10]** (CO2) (PO2)

(c) A camera is mounted at a point 3000 ft from the base of a rocket launching pad. If the rocket is rising vertically at 880 ft/s when it is 4000 ft above the launching pad, how fast must the camera elevation angle change at that instant to keep the camera aimed at the rocket? **[10]** (CO4) (PO2)

3. (a) Consider the function $f(x)=x^4-4x^3+10$ **[20]** (CO1) (PO2)

(i). Find the intervals on which $f(x)$ is increasing.

(ii). Find the intervals on which $f(x)$ is decreasing.

(iii). Find the open intervals on which $f(x)$ is concave up.

(iv). Find the interval on which $f(x)$ is concave down.

(v). Find the critical points and infection points, if any.

(b) Find the relative extrema of the function $f(x)=-x^3+6x^2+36x-60$. **[10]** (CO2) (PO2)

(c) Find the absolute maximum and absolute minimum values of the function $f(x)=2x^3-3x^2-12x+1$ on the interval $[-2,3]$, and determine where these values occur. **[10]** (CO2) (PO2)

4. (a) Write down the statement of Rolle's Theorem. Verify Rolle's Theorem for the function $f(x)=\ln\left(4+2x-x^2\right)$ on the interval $[-1,3]$. **[15]** (CO1) (PO2)

(b) An open box is to be made from a 16-inch by 30-inch piece of card board by cutting out squares of equal size from the four corners and bending up the sides. What size should the squares be to obtain a box with the largest volume? **[15]** (CO2) (PO2)

(c) (i). Find the second-order partial derivatives of $f(x,y)=4x^2-8xy^4+7y^5-3$. **[10]** (CO2) (PO2)

(ii). Let $u(w,x,y,z)=xe^{yw}\sin^2 z$ be a given function, then find $\frac{\partial^4 u}{\partial x\partial y\partial w\partial z}$ and evaluate $\frac{\partial u}{\partial w}(0,0,1,\pi)$.

The End