B.Sc. Eng. ME/IPE/1st Sem. Date: April 23, 2025

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF NATURAL SCIENCES

Semester Final Examination **Wineter Semester**: 2023-2024
Course Number: Math 4111 **Full Marks**: 120
Course Title: Modelling with calculus and ODE **Time**: 2 Hours

Please answer according to the order of the questions. Answer all the **4 (FOUR)** questions. The symbols have their usual meanings. Marks of each question and the corresponding CO and PO are written in the brackets.

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1. a) The formula for the power output P of a battery is (10) (CO2) (PO1)

$$P = VI - RI^2$$

where V is the electromotive force in volts, R is the resistance in ohms, and I is the current in amperes. Find the current that corresponds to a maximum value of P in a battery for which $V = 12$ volts and $R = 0.5$ ohm. Assume that a 15-ampere fuse bounds the output in the interval $0 \leq I \leq 15$. Could the power output be increased by replacing the 15-ampere fuse with a 20-ampere fuse? Explain.

b) Coughing forces the trachea (windpipe) to contract, which affects the velocity v of the air passing through the trachea. The velocity of the air during coughing is (10) (CO2) (PO1)

$$v = k(R - r)r^2, \quad 0 \leq r < R$$

where k is a constant, R is the normal radius of the trachea, and r is the radius during coughing. What radius will produce the maximum air velocity?

c) Using the second derivative test find the relative extrema of $f(x) = -3x^5 + 5x^3$. (10) (CO1) (PO1)

2. a) A farmer plans to fence a rectangular pasture adjacent to a river (see the **Figure Q2.a**. The pasture must contain 405,000 square meters in order to provide enough grass for the herd. No fencing is needed along the river. What dimensions will require the least amount of fencing? (10) (CO2) (PO1)

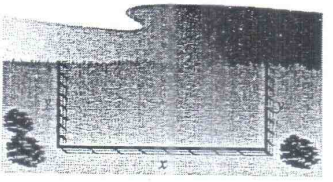

Figure Q2.a: A Rectangular Pasture Adjacent to a River

b) Use Newton's Method to approximate the zeros of $f(x) = 2x^3 + x^2 - x + 1$. Continue the iterations until two successive approximations differ by less than 0.0001. (10) (CO2) (PO2)

c) Find the area of the region bounded by the graph of $y = 2x^2 - 3x + 2$, the x-axis, and the vertical lines $x = 0$ and $x = 2$. (10) (CO2) (PO2)

3. a) Find the general solution the following differential equations: (10) (CO1) (PO1)

(i) $y(2xy+1)dx+x(1+2xy-x^3y^3)dy=0,$

(ii) $(D^4-81)x=0$, where $D\equiv\frac{d}{dt}$.

b) The motion of a mass attached to a spring with external force $f(t)=5\cos 4t$ is shown in **Figure Q3.b**, which satisfies the following initial value problem (IVP): (10) (CO3) (PO1)

$$\frac{1}{5}\frac{dx^2}{dt^2}+1.2\frac{dx}{dt}+2x=f(t),\ \text{with}\ x(0)=\frac{1}{2},\ x'(0)=0.$$

Apply undetermined coefficients method to solve the above-mentioned IVP.

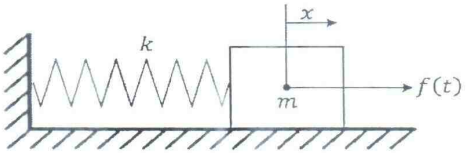

Figure Q3.b: A block-spring system with an external for $f(t)$.

c) Solve: $y''-4y'+4y=4x+8x^3$. (10) (CO3) (PO2)

4. a) Find the charge $q(t)$ on the capacitor in an *LRC*-series circuit in **Figure Q4.a** when $L=0.25$ henry(h), $R=10$ ohms (Ω), $C=0.001$ farad (f), $E(t)=0$, $q(0)=q_0$ coulombs (C), and $i(0)=0$. Write the linear second-order differential equation that satisfies the *LRC*-series circuit. Hence formulate the IVP and solve it. (10) (CO3) (PO1)

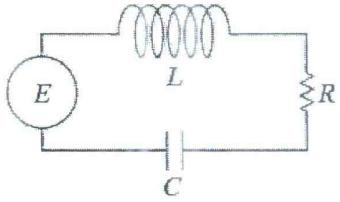

Figure Q4.a: *LRC*-series circuit

b) Apply variation of parameters method to solve $2y''-8y'+8y=(x+1)e^{2x}$. (10) (CO3) (PO2)

c) Solve: $x^2y''-7xy'+16y=0$ with $y(1)=0, y'(1)=1$. (10) (CO3) (PO1)