

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)DEPARTMENT OF NATURAL SCIENCE

Mid-Semester Examination

Summer Semester. A. Y. 2023-2024

Course Code.: Math 4221/4629

Full Marks: 120

Course Title: Mathematics III

Time: 2 hours

Answer all 4 (FOUR) questions. The symbols have their usual meanings. Programmable calculators are not allowed. Marks of each question and corresponding COs and POs are written in the brackets.

1. a) Explain the geometrical meaning of the scalar triple product of three vectors. (15)
Prove that in scalar triple product dot and cross can be interchanged. (CO1)
(PO1)
- b) Show that $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$ (15)
hence obtain the value of $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b})$. (CO1)
(PO1)
2. a) Find the values of (i) $\nabla \times \nabla \phi$ and (ii) $\nabla \cdot (\nabla \times \mathbf{A})$ (15)
(CO1)
(PO1)
- b) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at P (-1, -2, 1) in the direction of the vector $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$. Also find the maximum rate of change of ϕ at this point. (15)
(CO1)
(PO1)
3. a) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = xy\mathbf{i} + (x^2 + y^2)\mathbf{j}$ and C is the portion of the x-axis from (15)
 $x = 4$ to $x = 6$ and then the line $x = 6$ from $y = 0$ to $y = 20$. Is the force $\mathbf{F}$ (CO2)
conservative? (PO1)
- b) Find the flow of current through the surface S where S is the part of the plane (15)
 $x + 3y + 5z = 15$ which is located in the first octant, if the current density is (CO2)
 $\mathbf{J} = 10z\mathbf{i} + 10y\mathbf{j} + 3y\mathbf{k}$. (PO1)
4. a) State and prove Green's theorem in the plane. (15)
(CO3)
(PO1)
- b) Evaluate $\int_C (y - \sin x)dx + \cos x dy$ by using Green's theorem where C is the triangle (15)
(CO3)
(PO1)
formed by the straight lines $y=0$ from $(0,0)$ to $(\frac{\pi}{2}, 0)$, $x=\frac{\pi}{2}$ from $(\frac{\pi}{2}, 0)$ to $(\frac{\pi}{2}, 1)$
and $y=\frac{2x}{\pi}$ from $(\frac{\pi}{2}, 1)$ to $(0,0)$.