ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION SUMMER SEMESTER, 2023-2024

DURATION: 2 HOURS FULL MARKS: 120

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all 4 (four)** questions. Figures in the right margin indicate full marks of questions with corresponding COs and POs in parentheses.

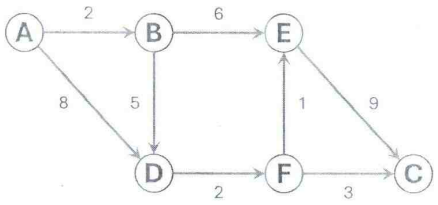

Figure 1: Directed Weighted Graph for Question 1.a and Question 1.b

1. a) Illustrate how Dijkstra's algorithm can be applied to find the shortest path from node **A** to node **C** of the directed weighted graph in Figure 1. — 10 (CO2) (PO1)

   b) Given the directed weighted graph in Figure 1, starting from vertex A, perform both *Breadth-First Search* (BFS) and *Depth-First Search* (DFS) to generate the respective directed spanning trees with a sequential illustration of the process from start to end. Visit the unvisited vertices in alphabetical order when multiple choices are available. — 5 + 5 (CO1) (PO1)

   c) "*Dijkstra's Algorithm will work only for directed and undirected graphs with positive weights*," explain with a suitable example. — 5 (CO1) (PO1)

   d) Apply Dirac's and Ore's theorem to the graph in Figure 1 and determine the presence of a Hamiltonian circuit while ignoring the directions. — 5 (CO3) (PO2)

   e) Explain the concept of *Relaxation* in Dijkstra's Algorithm with a suitable example. — 5 (CO1) (PO1)

2. a) Perform a two-step encryption on the message, "THIS MARKS THE END OF DISCRETE MATHEMATICS"; firstly, by using public key cryptography with a key value of $k = 5$, and then by using transposition cipher with $\sigma = \{1, 2, 3, 4\}$ and $\sigma(1) = 2, \sigma(2) = 4, \sigma(3) = 1$, and $\sigma(4) = 3$. Ignore spaces during the encryption process. — 5 + 5 (CO2) (PO1)

   b) Use Huffman Coding to generate the prefix codes of the characters in the string, "THIS MARKS THE END OF DISCRETE MATHEMATICS" Calculate the average number of bits required to represent each character using the generated prefix codes. — 15 + 5 (CO2) (PO1)

   [N.B.: Include spaces and consider arranging characters/sub-trees with the same probability in lexicographic order. For equal probabilities, assign the character/sub-tree that appears earlier as the left child in the next iteration.]

   c) How does Huffman coding resolve the issues associated with simple prefix codes? Explain with an example. — 5 (CO2) (PO1)

3. A manager at a pizza joint must decide how long to promise customers their orders will take as the number of deliveries, x, grows. The pizza shops delivery time for an order of x pizzas is modeled by $f(x) = 7x^2$ (minutes). Based on the delivery time of the joint in recorded history, Connie, a very conservative manager, says, "*Your pizza might take up to* $g(x) = x^3$ *minutes.*" On the other hand, Leo, an optimistic manager of the joint, says, "*Your pizza will take about* $h(x) = 10x^2$ *minutes.*" Based on this information, answer the following —

   a) Examine if Connie's and Leo's estimates provide valid upper bounds on the actual delivery time. Include a mathematical justification for each estimate. — 7 + 7 (CO2) (PO1)

   b) Determine whether Leo's estimate $h(x)$ is a tight bound for the actual delivery time $f(x)$. — 6 (CO2) (PO1)

   c) As the number of deliveries becomes very large, which estimate becomes less practical? Explain using the concept of growth rates. — 5 (CO2) (PO1)

4.
$$\frac{\sqrt{b^2 - 4ac}}{2ac - b} \quad (1)$$

Answer the following questions based on Equation 1 —

   a) Generate the corresponding rooted tree. — 5 (CO1) (PO1)

   b) Generate the corresponding *prefix* and *postfix* notations. — 5 + 5 (CO2) (PO1)

   c) Evaluate the *prefix* and *postfix* notations considering $a = 2, b = 5, c = 2$. — 5 + 5 (CO2) (PO1)