

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF NATURAL SCIENCES

Final Semester Examination

Summer Semester: 2023 - 2024

Course Code: MATH 4261

Full Marks: 120

Course Title: Mathematics II

Time: 2 Hours

There are 4 (four) questions. Answer **all** questions. Marks of each question and corresponding CO and PO are written in the brackets. Programmable calculators are not allowed. Do not write on this question paper. The Symbols have their usual meaning.

- 1.(a) State the Fundamental Theorem of Calculus involving definite integrals. [5] CO1
PO1

- (b) Use the method of substitution to find value of the following: [14] CO2
PO1

(i) $\int_0^1 x^3 \sqrt{1+3x^4} dx$; (ii) $\int_0^{\pi/4} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

- (c) The interest rates charged by Madison Finance on auto loans for used cars over a certain 6-month period in 2008 are approximated by the function [11] CO3
PO1

$$r(t) = -\frac{1}{12}t^3 + \frac{7}{8}t^2 - 3t + 12 \quad (0 \leq t \leq 6)$$

where t is measured in months and $r(t)$ is the annual percentage rate.

Calculate the average rate on auto loans extended by Madison over the 6-month period.

- 2.(a) An efficiency study conducted for Elektra Electronics showed that the rate at which Space Commander walkie-talkies are assembled by the average worker t hours after starting work at 8:00 a.m. is given by the function [11] CO3
PO1

$$f(t) = -3t^2 + 12t + 15 \quad (0 \leq t \leq 4).$$

Determine how many walkie-talkies can be assembled by the average worker in the first hour of the morning shift.

- (b) Find the area under $f(x) = x^{1/3} + 5$ over the interval $x = 1$ to $x = 8$. [8] CO2
PO1

- (c) The demand equation for a product is $p_d = 60 - \frac{50q}{\sqrt{q^2 + 3600}}$ and the supply equation is $p_s = 10 \ln(q + 20) - 26$. Determine consumers' surplus and producers' surplus under market equilibrium. Round your answers to the nearest integer. [11] CO3
PO1

- 3.(a) Derive the Newton-Raphson's method for finding the root of a nonlinear equation $f(x) = 0$. [5] CO1
PO1
- (b) Apply Newton-Raphson's method to compute the real root of the equation $x \log_{10} x - 1.2 = 0$, correct to three decimal places. [12] CO2
PO1
- (c) Find the real root of $x^3 - 2x - 5 = 0$ using the bisection method, accurate to three decimal places. [13] CO3
PO1
- 4.(a) State and explain the fourth order Runge-Kutta method for solving first-order differential equations. [5] CO1
PO1
- (b) Apply the fourth order Runge-Kutta method to solve $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$. [12] CO2
PO1
Take step size $h = 0.1$ and determine approximations to $y(0.1)$ and $y(0.2)$ correct to four decimal places.
- (c) The following table gives the population of a town during the last six censuses. [13] CO3
PO1
Estimate the increase in the population during the period from 1946 to 1948 using Newton's formula for forward interpolation:

Year:	1911	1921	1931	1941	1951	1961
Population: (in thousands)	12	15	20	27	39	52