B.Sc. Eng. (CEE), 2nd Sem. Date: July 8, 2025

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF NATURAL SCIENCES

Mid Semester Examination **Summer Semester**: 2023-2024
Course Number: MATH 4253 **Full Marks**: 120
Course Title: Vector Algebra, Vector Calculus and ODE **Time**: 2 Hours

Answer all the 4 (**FOUR**) questions. The symbols have their usual meanings. Marks of each question and the corresponding CO and PO are written in the brackets.

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1. a) Forces of magnitude 3 and 2 in the directions $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ and $2\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$ respectively act on a particle which is displaced from the point $(2,-1,-3)$ to $(5,-1,1)$. Find the work done by the force. **(8)** (CO1) (PO1)

b) Examine whether the four points whose position vectors are $3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$, $6\hat{\imath} + 3\hat{\jmath} + \hat{k}$, $5\hat{\imath} + 7\hat{\jmath} + 3\hat{k}$ and $2\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$ are coplanar or not. **(10)** (CO3) (PO1)

c) Solve the simultaneous vector equations for $\underline{x}$ and $\underline{y}$; $\underline{x} + \underline{c} \times \underline{y} = \underline{a}$ and $\underline{y} + \underline{c} \times \underline{x} = b$, where $\underline{c} \neq 0$ **(12)** (CO3) (PO1)

2. a) Find the equation of a straight line which passes through two given points A and B have position vector $\underline{a}$ *and* $\underline{b}$ with respect to an origin o **(10)** (CO1) (PO1)

b) Find the directional derivative of $f(x,y,z) = x^2y^2z^2$ at the point $(1,1,-1)$ in the direction of the tangent to the curve $x = e^t, y = 2\sin t + 1, z = t - \cos t,$ *at* $t = 0$. **(10)** (CO2) (PO1)

c) A fluid motion is given by $\underline{v} = (ysinz - sinx)\boldsymbol{i} + (xsinz + 2yz)\boldsymbol{j} + (xycosz + y^2)\boldsymbol{k}$: Is the motion irrational? If so, find the velocity potential. **(10)** (CO2) (PO1)

3. a) Find the differential equation of all conics whose axes coincide with the axes of co-ordinates. **(10)** (CO3) (PO1)

b) Solve; $y^2dx + (xy + x^2)dy = 0$ **(10)** (CO1) (PO1)

c) For steady heat flow through the wall of a hollow sphere of inner and outer radii r_1 *and* r_2 respectively, the temperature u at a distance $r(r_1 < r < r_2)$ from the center of the sphere is given by **(10)** (CO3) (PO1)

$$\frac{d^2u}{dr^2} + 2\frac{du}{dr} = 0$$

If u_1 *and* u_2 are the temperatures at inner and outer surfaces of the sphere, find u in terms of r.

4. a) Solve: $\frac{dy}{dx} = \frac{x+y+7}{2x+2y+3}$ (8) (CO1) (PO1)

b) Solve: $\frac{dy}{dx} - 2y\,tanx = y^2 tan^2 x$ (10) (CO1) (PO1)

c) Find the integrating factor and hence solve: (12) (CO3) (PO1)

$$(xy^2 - e^{\frac{1}{x^3}})\,dx - x^2 y dy = 0$$