BBA in TM, 2nd Sem. Date: July 8, 2025

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF NATURAL SCIENCES

Mid-Semester Examination — Summer Semester, A. Y.: 2023-2024
Course No. : **MATH 4261** — **Time: 2.0 hours**
Course Title : **Mathematics II** — **Full Marks: 120**

Please answer according to the order of the questions. **Answer all of the 4/4 (Four) questions.** The symbols have their usual meaning. Marks of each question and the corresponding CO and PO are written in the brackets.

1. a) Marcy works at the B & O department store, where on a weekday, she is paid \$6 per hour for the first eight hours and \$9 per hour for overtime. You may verify that the function $f(x) = \begin{cases} 6x \; ; \; 0 \le x \le 8 \\ 9x - 24 \; ; \; 8 < x \end{cases}$ gives Marcy's earnings on a weekday in which she worked x hours. Test the continuity of the function at $x = 8$ using graph. **(5)** (CO1) (PO1)

b) If $f(x) = \begin{cases} \sqrt{2-x} & if\, x < 2 \\ x^3 + k(x+1) & if\, x \ge 2 \end{cases}$, determine the value of the constant k for which $\lim_{x \to 2} f(x)$ exists. **(10)** (CO2) (PO1)

c) Find values of the constants k and m, if possible, that will make the function f continuous everywhere. **(15)** (CO3) (PO1)

$$f(x) = \begin{cases} x^2 + 5, & x > 2 \\ m(x+1) + k, & -1 < x \le 2. \\ 2x^3 + x + 7, & x \le -1 \end{cases}$$

2. a) The demand function for the Sicard wrist watch is given by $d(x) = \frac{50}{0.01x^2+1} (0 \le x \le 20)$, where x (thousand) is the quantity demanded per week and $d(x)$ is the unit price in dollars. Find $d'(x)$. Hence compute $d'(5)$, $d'(10)$ and $d'(15)$ and interpret your results. **(8)** (CO1) (PO1)

b) If $y = a\,cos(lnx) + bsin(lnx)$, then applying Leibnitz's Theorem find the value of $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n$. **(10)** (CO2) (PO1)

c) Apply L' Hospital's Rule to evaluate the following limit **(12)** (CO3) (PO1)

$\lim_{x \to 0} \left[\frac{1}{x^2} - \frac{1}{sin^2 x}\right]$.

3. **a)** State Euler's Theorem. Hence verify for the function $u(x, y) = \frac{x^{1/6}+y^{1/6}}{x^{1/7}+y^{1/7}}$. **(10)** (CO1) (PO1)

b) If $V = \sqrt{x^2 + y^2 + z^2}$ then show that $V_{xx} + V_{yy} + V_{zz} = 2/V$. **(8)** (CO2) (PO1)

c) If $f(x) = x^3 - 3x^2 + 1$ then use the first and second derivatives of f to determine the intervals on which f is increasing, decreasing, concave up, and concave down. Also locate the inflection points. **(12)** (CO3) (PO1)

4. **a)** Integrate the following: **(12)** (CO1) (PO1)

(i) $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}} dx$; (ii) $\int \frac{1}{\sqrt{sin^5 x \, cos^7 x}} dx$; (iii) $\int e^x(tanx - logcosx)\, dx$

b) Find the relative extrema of $f(x) = 3x^5 - 5x^3$ **(10)** (CO2) (PO1)

c) Expand $f(x) = 2x^3 + 7x^2 - x - 1$ in powers of $(x - 2)$. **(8)** (CO3) (PO1)