(7)

B.Sc. Eng. (ME/IPE)/ 3rd Sem.

Date: January 14, 2025

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF NATURAL SCIENCES

Semester: Mid Term Examination
Course No.: MATH 4311
Course Title: Vector Analysis, Multivariable Calculus and Complex Variables

Winter Semester: 2024-2025
Full Marks: 120
Time: 2.0 Hours

- There are **4 (Four)** sets of questions. Answer **all** of them.
- The figures in the right margin indicate full marks. **CO**s and **PO**s are also specified in the right margin of the questions.
- The symbols have their usual meaning.

			CO	PO	Full Marks
1.	a)	Find each of the indicated roots of $(-1+i)^{\frac{1}{4}}$ using De Moivre's theorem and locate them graphically.	1	2	[10]
	b)	Show that $\cos 5\theta$ can be expressed as polynomials in $\cos\theta$ and $\sin\theta$	3	2	[10]
	c)	Represent graphically the set of values of z for which $\left\|\frac{z-3}{z+3}\right\| = 2.$	2	2	[10]
2.	a)	Verify that the real and imaginary parts of the function $f(z) = z^2 + 5iz - 3 - i$ satisfy the Cauchy–Riemann equations	1	2	[8]
	b)	Show that the function $u(x,y) = x^3 - 3x^2y - 3xy^2 + y^3$is harmonic. Then find a function $v(x,y)$ such that $f(z) = u(x,y) + iv(x,y)$ is analytic.	3	2	[12]
	c)	Consider the transformation $w = (1+i)z + 2 - i$. Determine the region in the w plane to which the rectangular region bounded by the lines $x = 0, y = 0, x = 1, y = 2$ in the z plane is mapped under this transformation.	2	2	[10]
3.	a)	Consider three electric field vectors: $\boldsymbol{E_1} = i + 2j + 3k,\ \boldsymbol{E_2} = 4i + 5j + 6k,\ \boldsymbol{E_3} = 7i + 8j + 9k$ Check if these vectors are linearly dependent or independent. If possible, find a relation among them.	2	2	[10]

b) Find the orthogonal projection of $\boldsymbol{v} = i + j + k$ on $\boldsymbol{b} = 2i + 2j$, and then find the vector component of $\boldsymbol{v}$ orthogonal to $\boldsymbol{b}$. 2 2 [10]

c) Define reciprocal set of vectors. Find a set of vectors reciprocal to the set 1 1 [10]

$\boldsymbol{v_1} = i + 2j + 3k,\ \boldsymbol{v_2} = 4i + 5j + 6k,\ v_3 = 7i + 8j + 9k$

4 a) State the Frenet-Serret formulae. 1 1 [5]

b) Consider the curve C represented by the vector valued function 2 2 [15]

$$\mathbf{r}(t) = e^t \cos t\,\mathbf{i} + e^t \sin t\,\mathbf{j} + e^t\mathbf{k}$$

i) Find $\mathbf{T}(t), \mathbf{N}(t)$ and $\mathbf{B}(t)$ for the curve C at $t = \pi$.
ii) Find the curvature and torsion of curve C at $t = \pi$.
iii) Find the arc length of the curve C from $t = 0$ to $t = 10$.

c) Write down the vector function of the surface S: 2 2 [10]
The plane $3x + 2y + z = 6$ bounded by the coordinate planes and its boundary. Hence derive the unit normal $\hat{n}$ and elementary surface dS to the surface S.