Libry

(4)

B.Sc. Eng. (ME/IPE)/1st Sem. **Date: 06/04/2026 (Afternoon)**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

DEPARTMENT OF NATURAL SCIENCES

Final Examination **Winter Semester**: 2024-2025
Course Code: MATH 4111 **Full Marks**: 120
Course Title: Modelling with calculus and ODE **Time**: 2 Hours

Please answer according to the order of the questions. Answer all the **4 (FOUR)** questions. The symbols have their usual meanings. Marks of each question and the corresponding CO and PO are written in the brackets.

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1. a) Find the followings: $(\mathrm{i})\int \frac{x^3}{\sqrt{1+x^2}}dx$, $(\mathrm{ii})\int x^3 \tan^{-1} x \; dx$. **(10)** (CO1) (PO1)

b) Compute the following integrals: **(10)** (CO1) (PO1)

$(i)\int_0^{\pi/2} \frac{1}{1+\sqrt[5]{\tan x}}dx$, $(\mathrm{ii})\int_0^{\pi/8} \sin^2(8x)\cos^4(4x)\,dx$, $(\mathrm{iii})\int_0^{\infty} e^{-x^2}x^{10}\,dx$.

c) (i) State Walli's formula. Hence compute the integral: $\int_0^{\pi/6} \sin^{24}(3x)dx$. **(10)** (CO1) (PO1)

(ii) Justify whether the improper integral converge or diverge $\int_{-\infty}^{\infty} \frac{1}{1+x^2}dx$..

2. a) Let R be the region bounded by the loop of the curve $a^2y^2 = x^3(2a-x)$. **(10)** (CO2) (PO2)
Compute the area of the region R.

b) Let $y = f(x)$ be a curve. (i) Derive an integral formula to calculate the length of the said curve. Hence (ii) find the length of the curve $y = \sqrt{9-x^2}$, $0 \le x \le 3$ illustrated in **Fig. Q2(b),** (iii) verify your answer by noting that the curve is a part of a circle. **(10)** (CO2) (PO2)

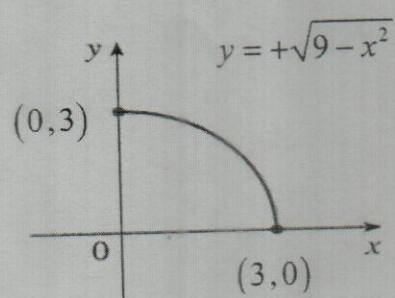


Fig. Q2(b)

c) Apply integral technique to find the volume of the solid generated by revolving the regions bounded by $y = 4-x^2$, $y = -5$ about the line $y = -5$. **(10)** (CO2) (PO1)

3. a) Define homogeneous differential equation. (1+7) (CO1) (PO1)
Solve: $(D^2 + 2D + 2)y = 0; y(0) = 0, y'(0) = 0.$

b) Find the general solution of the differential equation $y'' - 2y' + 2y = x + e^x cosx.$ (10) (CO1) (PO1)

c) Solve: $\frac{d^2y}{dx^2} - y = x\, sin3x + cosx$ (12) (CO1) (PO1)

4. a) Apply the method of factorization of differential operators and solve the differential equation $[(x + 1)D^2 + xD - D - 2]y = 0.$ (12) (CO2) (PO2)

b) Define convergent and divergent sequences with example. (2+16) (CO2) (PO2)
Apply the p-series test to determine whether the series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots \infty$ is convergent or divergent. State the condition on p for convergence and divergence.

*** The End ***