Program: B. Sc. Eng. (ME/IPE), $2^{nd}$ Semester | Date: September 16, 2025

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# ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

## DEPARTMENT OF NATURAL SCIENCES

Final Semester Examination | Summer Semester: 2023 - 2024
Course Code: Math 4211 | Full Marks: 120
Course Title: PDE, Special Functions, Laplace and Fourier Analysis | Time: 2 Hours

There are 4 **(four)** questions. Answer **all** questions. Marks of each question and corresponding CO and PO are written in the brackets. Programmable calculators are not allowed. Do not write on this question paper. The Symbols have their usual meaning.

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**1.(a)** Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$ **[20]** CO1 PO1

**(b)** Apply method of Frobenious to solve in series the differential equation (Subscripts indicate the order of the differentiation) **[10]** CO2 PO1

$$2x^2y_2 - xy_1 + (1-x^2)y = 0.$$

**2.(a)** Show that $J_{\frac{5}{2}}(x) = \sqrt{\left(\frac{2}{\pi x}\right)}\left[\frac{(3-x^2)}{x^2}\sin x - \frac{3\cos x}{x}\right].$ **[10]** CO1 PO1

**(b)** (i) Prove that $nP_n(x) = (2n-1)xP_{n-1}(x) - (n-1)P_{n-2}(x),$ **[10]** CO2 PO1

and hence apply the formula to find $P_3(x)$.

**(ii)** Apply suitable technique to show that **[10]** CO2 PO1

$$\int_{-1}^{1} x^2 P_n(x)P_{n-2}(x)dx = \frac{2n(n-1)}{(2n+1)(2n-3)(2n-1)}.$$

**3.(a)** (i) Determine the Fourier Integral of **[10]** CO2 PO1

$$f(t) = \begin{cases} 1+t, & |t| \le 1 \\ 0, & |t| > 1 \end{cases}.$$

Hence apply Fourier Integral to evaluate the integral $\int_0^\infty \left(\frac{\sin u}{u}\right)^2 du.$

**(ii)** Find the Fourier sine and cosine transform of $f(x) = \begin{cases} 1, & 0 < x < a \\ 0, & \text{otherwise} \end{cases}$ **[10]** CO1 PO1

**(b)** Derive the system of differential equations describing the straight-line vertical motion of the coupled springs shown in equilibrium in **FigQ3(b)**. Apply the Laplace transform to solve the system when $k_1 = 1, k_2 = 1, k_3 = 1$ $m_1 = 1, m_2 = 1$ and $x_1(0) = 0, x_1'(0) = -1, x_2(0) = 0, x_2'(0) = 1.$ **[10]** CO2 PO1

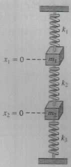


**Fig. Coupled Springs in Q3(b))**

4.(a) (i) Determine **real** and, **complex** form of Fourier series of sawtooth wave function $f(x)$ given by **Fig Q4a(i).** **[10]** CO1 PO1

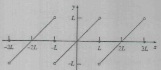


**Fig. Q4a(i)**

**(ii)** Compute the Fourier sine and cosine series of the following function $f(t)$ defined in Fig.Q4a(ii): **[10]** CO1 PO1

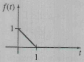


**Fig. Q4a(ii)**

**(b)** Apply finite Fourier transform to solve the one-dimensional heat conduction equation $\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$, subject to the boundary conditions in Figure Q4(b) and the initial condition $T(x,0) = 2x$, where $0 < x < 4$. **[10]** CO2 PO1

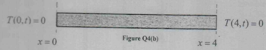


Figure Q4(b)