ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

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SEMESTER FINAL EXAMINATION SUMMER SEMESTER, 2023-2024

DURATION: 2 HOURS FULL MARKS: 120

CSE 4203: Discrete Mathematics

Programmable calculators are not allowed. Do not write anything on the question paper. Answer all 4 (four) questions. Figures in the right margin indicate full marks of questions with corresponding COs and POs in parentheses.

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1. a) An online company decided to give a 24% discount for their first 100 orders. However, only those orders are eligible whose order numbers are congruent to -1 **mod** 13. (The company selected the number 13 as a symbolic reference to August 5: $5+8 = 13$). If the order numbers start from 1, determine: 3 + 3 (CO3) (PO1)

   i. How many people will get this discount?

   ii. What are the order numbers of those customers?

   b) Define Pigeonhole Principle. How many integers must you choose from the set $\{1, 2, \ldots, 100\}$ to guarantee that at least 5 of them have the same last digit? 1 + 4 (CO2) (PO1)

   c) Let a, b, k, m be integers with $k \geq 1$ and $m \geq 2$. If $a \equiv b \pmod{m}$, then show that 10 (CO3) (PO1)

$$a^k \equiv b^k \pmod{m}.$$

   d) At the annual village fair, two farmers bring baskets of mangoes: 7 + 3 (CO3) (PO1)

   - Farmer **X** baskets each contain **1001 mangoes**.
   - Farmer **Y** baskets each contain **100001 mangoes**.

   The chairman wants to prepare a special display of mangoes by combining those from both farmers. He insists that the total number of mangoes in this display must be expressible in the form

$$1001m + 100001n$$

   for some integers m and n. Based on this scenario answer following questions.

   i. Find integers m and n that make the **smallest possible positive number** of mangoes.

   ii. Explain what this smallest number represents in the context of the village fair.

2. a) Consider the following system of congruences and answer the following questions. 10 + 5 (CO3) (PO1)

$$x \equiv 7 \pmod{9}, \qquad x \equiv 4 \pmod{12}, \qquad x \equiv 16 \pmod{21}.$$

   i. Show whether the system is consistent, and if so find all integer solutions.

   ii. List all solutions x with $0 \leq x < 1000$.

   b) Using Fermat's Little Theorem, compute the following modulo 23. 5 (CO3) (PO1)

$$7^{2025} + 5^{2025} \pmod{23}.$$

   c) Decrypt the ciphertext **MNAESOTPCRE**, which was produced by encrypting a plaintext message using a block transposition cipher with blocks of four letters. The permutation σ of $\{1, 2, 3, 4\}$ is defined by: 2 + 10 (CO3) (PO1)

$$\sigma(1) = 3, \quad \sigma(2) = 1, \quad \sigma(3) = 4, \quad \sigma(4) = 2.$$

   i. Divide the ciphertext into blocks corresponding to the transposition cipher.

   ii. Using the given permutation, decrypt each block to recover the original plaintext message.

3. a) Define **Big-O notation**. Clearly explain its formal mathematical definition and meaning in algorithm analysis. For the function $f(n)$ determine the **Big-O complexity**. Show all steps, including simplification using logarithmic properties and justification of the dominant term. 2 + 8 (CO2) (PO1)

$$f(n) = n^2 \cdot \log(n^2 + 1) + n^2 \cdot \log n!$$

b) In a small town in Bangladesh, a network of streets is represented by a simple undirected graph $G = (V, E)$, where intersections are vertices and streets are edges. The road network has 8 intersections. The number of streets incident at each intersections is given as follows: 4 × 3 (CO4) (PO1)

$$i_1 = 3,\ i_2 = 3,\ i_3 = 2,\ i_4 = 2,\ i_5 = 2,\ i_6 = 2,\ i_7 = 3,\ i_8 = 2$$

Answer the following question for the graph G.

i. Determine the total number of edges in the graph.

ii. Determine whether G can be connected. Justify your answer.

iii. Is it possible for G to have an Eulerian path or Eulerian circuit? Explain.

c) Consider the two graphs described below. Determine if they are isomorphic. Justify your answer by either providing an isomorphism (a bijection that preserves adjacency) or by demonstrating a lack of a preserved graph invariant (a property that must be the same for both graphs if they are isomorphic). 10 (CO4) (PO1)

Graph G_1: This is the wheel graph W_5. It consists of a central vertex adjacent to all five vertices of a cycle graph C_5.

Graph G_2: This is a graph with vertex set $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and edge set E defined by the following adjacency lists:

- v_1 is adjacent to $\{v_2, v_3, v_4, v_5, v_6\}$
- v_2 is adjacent to $\{v_1, v_3, v_4\}$
- v_3 is adjacent to $\{v_1, v_2, v_5\}$
- v_4 is adjacent to $\{v_1, v_2, v_6\}$
- v_5 is adjacent to $\{v_1, v_3, v_6\}$
- v_6 is adjacent to $\{v_1, v_4, v_5\}$

4. a) A tech company is designing a **multi-level server network** where each server (internal node) can connect to up to m other servers (children). The network is structured as a **full m-ary tree** with a height of 4 and exactly 81 endpoint devices (leaves) connected at the lowest level. 5 + 5 (CO4) (PO1)

i. Determine the **upper and lower bounds for** m based on this setup.

ii. If the network is designed to be **perfectly balanced**, find the exact value of m.

b) A binary expression tree is a rooted, ordered tree in which: 3 × 5 (CO4) (PO1)

- Every internal node is labeled with a binary operator (+, −, ×, /), and
- Every leaf node is labeled with an operand (variable or constant).

You are given the following **postfix expression**:

$$P = a\ b\ c\ \times\ d\ +\ e\ f\ -\ g\ \times\ +\ h\ i\ +\ \times$$

i. Construct the corresponding **ordered rooted binary expression tree** from the given postfix expression.

ii. Write the equivalent **prefix** and **infix** expressions obtained by traversing this tree in preorder and inorder respectively.

iii. If the variables have the following values:

$$a = 2,\ b = 3,\ c = 4,\ d = 5,\ e = 1,\ f = 6,$$

evaluate the expression step by step using the postfix notation.